International Journal of Engineering, Science and Mathematics
Vol.9 Issue 1,January 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

MINIMAL TOTAL UNIDOMINATING FUNCTIONS WITH MAXIMUM WEIGHT OF A PATH

V.ANANTHA LAKSHMI[1]* B.MAHESWARI[2]

[1]Lecturer, Department of Mathematics, P.R.Govt. College (A), Kakinada-533004, Andhra Pradesh, India

[2]Professor(Retd.), Department of Applied Mathematics, S.P.MahilaVisvavidyalayam(Women's University), Tirupati-517502, Andhra Pradesh, India

ABSTRACT

The upper unidomination number of a path and the number of minimal unidominating functions of a path with maximum weight were found in [12]. The upper total unidomination number of a path was found in [13]. In this paper the number of minimal total unidominating functions of a path with maximum weight isfound.

1.INTRODUCTION

Graph Theory plays a vital role in several areas of computer science such as switching theory, logical design, artificial intelligence, formal languages, computer graphics, compiler writing, information organization retrieval etc.

In Graph Theory, one of the rapidly growing area of research is the theory of domination which was introduced by Berge [2] and Ore [7]. Total dominating sets were introduced by Cockayane, Dawes and Hedetniemi [3]. Some results on total domination can be seen in [1].

Domination and its properties have been extensively studied by T.W.Haynes et.al.in [8], [9]. Domination in graphs have applications to several fields such as School bus routing, Computer communication networks, Facility location problems, Locating radar stations problem etc.

Recently dominating functions in domination theory have received much attention. Hedetniemiet.al. [6] introduced the concept of dominating functions and the concept of total dominating functions, was introduced by Cockayne et.al. [4]. Properties of minimal dominating functions are studied in [5]. The concept of total unidominating function was introduced by the authors in [10]. Minimal total unidominating functions and upper total unidomination number were introduced in [11]. The upper total unidominationnumber of a path was found in [13].

In this paper the minimal total unidominating functions and upper total unidomination number of a path are discussed and the number of minimal total unidominating functions with maximum weight is found and the results obtained are illustrated.

2.UPPER TOTAL UNIDOMINATION NUMBER OF A PATH

In this section the minimal total unidominating functions of a patharediscussed and also the number of minimal total unidominating functions withmaximum weight is found.

First the concepts of total unidominating function, minimal total unidominating functions and upper total unidomination number are defined as follows.

Definition 2.1: Let $G(V,E)$ be a connected graph. A function $f: V \rightarrow \{0,1\}$ is said to be a **total unidominating function,** if

$$\sum_{u \in N(v)} f(u) \geq 1 \ \forall\, v \in V \ and\ f(v) = 1,$$

$$\sum_{u \in N(v)} f(u) = 1 \ \forall\, v \in V \ \ and\ \ f(v) = 0,$$

where $N(v)$ is the open neighbourhood of the vertex v.

Definition 2.2: Let $G(V,E)$ be a connected graph.A total unidominating function $f: V \rightarrow \{0,1\}$ is called a **minimal total unidominating function** if for all $g < f$, g is not a total unidominating function.

Definition 2.3: The **upper total unidomination number** of a connected graph $G\ (V,E)$ is defined asmax$\{f(V)/f$ is a minimal total unidominating function$\}$. It is denoted by$\Gamma_{tu}(G)$.

International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

We need the following theorem published by the authors and the proof can be found in [13]

Theorem 2.1: The upper total unidomination number of a path P_n is

$$\Gamma_{tu}(P_n) = \begin{cases} 2 & if\ n = 2, \\ \left\lfloor \frac{5n}{7} \right\rfloor & if\ n > 2. \end{cases}$$

The number of minimal total unidominating functions with maximum weight is found in the following theorem.

Theorem 2.2: The number of minimal total unidominating functions of P_n with maximum weight is

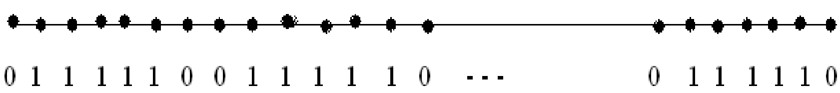
$$\begin{cases} 1 & when\ n \equiv 0(mod\ 7), \\ \left\lfloor \frac{n}{7} \right\rfloor \left\lceil \frac{n}{7} \right\rceil & when\ n \equiv 1(mod\ 7), \\ \left\lfloor \frac{2n}{7} \right\rfloor & when\ n \equiv 2(mod\ 7), n \neq 2, \\ 1 & when\ n = 2, \\ 2 & when\ n \equiv 3(mod\ 7), \\ \frac{1}{2}\left\lceil \frac{n}{7} \right\rceil \left\lceil \frac{3n}{7} \right\rceil + \frac{1}{6}\left\lceil \frac{n}{7} \right\rceil \left\lfloor \frac{n}{7} \right\rfloor \left(\left\lfloor \frac{n}{7} \right\rfloor - 1 \right) & when\ n \equiv 4(mod\ 7), \\ \left\lceil \frac{n}{7} \right\rceil + \frac{1}{2}\left\lceil \frac{n}{7} \right\rceil \left\lfloor \frac{n}{7} \right\rfloor + \left\lfloor \frac{n}{7} \right\rfloor & when\ n \equiv 5(mod\ 7), \\ \left\lceil \frac{n}{7} \right\rceil + 1 & when\ n \equiv 6(mod\ 7). \end{cases}$$

Proof: Let P_n be a path with vertex set $V = \{v_1, v_2, \ldots, v_n\}$.

Now we find the number of minimal total unidominating functions with maximum weight in the following seven cases.

Case 1: Let $n \equiv 0(mod\ 7)$.

The minimal total unidominatingfunction f defined in Case 1 of Theorem 2.1 is given by



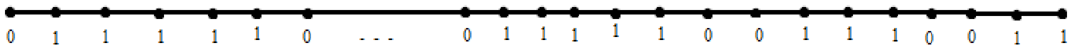
The functional values of f are $01111100111110 - - - 0111110$.

International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Take $a - 0111110$. Then the functional values of f are in the pattern of $aaa \ldots a$. These letters $aaa \ldots a$ can be arranged in one and only one way. Therefore there is only one minimal total unidominating function with maximum weight $\left\lfloor \frac{5n}{7} \right\rfloor$.

Case 2: Let $n \equiv 1(mod\ 7)$.

The minimal total unidominating function f defined in Case 2 of Theorem 2.1 is given by



The functional values of f are $0111110 \ldots 011111001110011$.

Take $a - 0111110$, $c - 01110$. Then the functional values of f are in the pattern of $aaa \ldots ac011$. As there are $\frac{n-8}{7}$ $a's$ and one c, these letters $a's$ and c can be arranged in $\frac{\left(\frac{n-1}{7}\right)!}{\left(\frac{n-8}{7}\right)!} = \frac{n-1}{7}$ ways. Therefore there are $\frac{n-1}{7}$ minimal total unidominating functions.

We further investigate some more minimal total unidominating functions of P_n with maximum weight in the following way.

Define a function $f_1 : V \to \{0,1\}$ by

$$f_1(v_i) = \begin{cases} 1 & for\ i \equiv 0,1,2,5,6(mod\ 7)\ i \neq n, \\ 0 & for\ i \equiv 3,4(mod\ 7), \end{cases}$$

and $f_1(v_n) = 0$.

As in Theorem 2.1 we can show that f_1 is a minimal total unidominating function. Also

$$\sum_{u \in V} f_1(u) = \underbrace{1+1+0} + \underbrace{0+1+1+1+1+1+0} + \cdots + \underbrace{0+1+1+1+1+1+0}$$
$$+ \underbrace{0+1+1+1+0} = 2 + \frac{5(n-8)}{7} + 3 = \frac{5n-5}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function is given by

International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

The functional values of f_1 are $1100111110 - - - 011111001110$.

Take a – 0111110, b – 011110, c – 01110.

Then the functional values of f_1 are in the pattern of $110aaa \dots ac$. In similar lines as above we can see that there are $\frac{n-1}{7}$ minimal total unidominating functions.

Define another function $f_2 : V \to \{0,1\}$ by

$$f_2(v_i) = f(v_i) \quad for\ all\ \ v_i \in V, i \neq n-7, n-9,$$

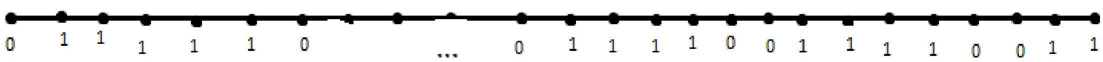
and $f_2(v_{n-7}) = 1,\ f_2(v_{n-9}) = 0,\ n \geq 15.$

We can see that this is a minimal total unidominating function.

Now $\sum_{u \in V} f_2(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+1+0}+$

$$\underbrace{0+1+1+1+1+0}+\underbrace{0+1+1} = \frac{5(n-15)}{7} + 10 = \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function is given by



The functional values of f_2 are $01111110 \dots 0111110011110011110011$.

Take $a - 0111110$ and $b - 011110$.Then $f_2$is in the pattern of $aaa \dots abb011$.These letters a's and b's can be arranged in $\frac{\left(\frac{n-1}{7}\right)!}{\left(\frac{n-15}{7}\right)!.2!} = \frac{\left(\frac{n-1}{7}\right)\left(\frac{n-8}{7}\right)}{2}$ ways.

Therefore there are $\frac{\left(\frac{n-1}{7}\right)\left(\frac{n-8}{7}\right)}{2}$ minimal total unidominating functions.

Define another function $f_3 : V \to \{0,1\}$ by

$$f_3(v_i) = f_1(v_i) \ \ for\ all\ v_i \in V, i \neq n-4, n-6$$

And $f_3(v_{n-4}) = 1,\ f_3(v_{n-6}) = 0,\ n \geq 15.$

This is also a minimal total unidominating function.

$$\text{Now} \sum_{u \in V} f_3(u) = \underbrace{1+1+0}+\underbrace{0+1+1+1+1+1+0}+\cdots$$

$$+\underbrace{0+1+1+1+1+0}+\underbrace{0+1+1+1+1+0}$$

$$= 2 + \frac{5(n-15)}{7} + 4 + 4 = \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function is given by

The functional values of f_3 are $1100111110 \ldots 01111100111100 11110$.

Take $a - 0111110$ and $b - 011110$. Then f_3 is in the pattern of $110aaa \ldots abb$.

These letters $a's$ and $b's$ can be arranged in $\frac{\left(\frac{n-1}{7}\right)!}{\left(\frac{n-15}{7}\right)!.2!} = \frac{\left(\frac{n-1}{7}\right)\left(\frac{n-8}{7}\right)}{2}$ ways.

Therefore there are $\frac{\left(\frac{n-1}{7}\right)\left(\frac{n-8}{7}\right)}{2}$ minimal total unidominating functions.

Thus there are $\frac{n-1}{7} + \frac{n-1}{7} + \frac{\left(\frac{n-1}{7}\right)\left(\frac{n-8}{7}\right)}{2} + \frac{\left(\frac{n-1}{7}\right)\left(\frac{n-8}{7}\right)}{2} = \left\lfloor \frac{n}{7} \right\rfloor . \left\lceil \frac{n}{7} \right\rceil$ minimal total unidominating functions with maximum weight $\left\lfloor \frac{5n}{7} \right\rfloor$.

Case 3: Let $n \equiv 2(mod\ 7)$.

The minimal total unidominating function f defined in Case 3 of Theorem 2.1 is given by

The functional values of f are $0111110 \ldots 0111110011110011$.

Take $a - 0111110$, $b - 011110$. Then f is in the pattern of $aaa \ldots ab011$. These letters $aaa \ldots ab$ can be arranged in $\frac{n-9}{7} + 1 = \frac{n-2}{7}$ ways.

Therefore there are $\frac{n-2}{7}$ minimal total unidominating functions.

Now as per the discussion in Case 2 we obtain some other minimal total unidominating functions.

Define a function $f_1: V \to \{0,1\}$ by

$$f_1(v_i) = \begin{cases} 1 & for\ i \equiv 0,1,2,5,6(mod\ 7)\ i \neq n, \\ 0 & otherwise. \end{cases}$$

On similar lines as in Theorem 2.1 we can show that f_1 is a minimal total unidominating function.

Now $$\sum_{u \in V} f_1(u) = \underbrace{1+1+0}+\underbrace{0+1+1+1+1+1+0}+\cdots+$$

$$\underbrace{0+1+1+1+1+1+0}+\underbrace{0+1+1+1+1+0} = 2+\frac{5(n-9)}{7}+4 = \frac{5n-3}{7}$$

$$= \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function f_1 is given by

That is the functional values of f_1 are 1100111110 ... 0111110011110.

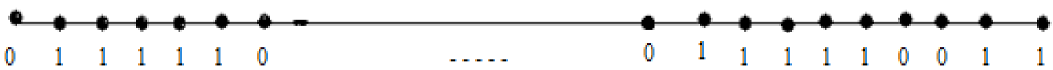
That is f_1 is in the pattern of $110aa \ldots ab$.

Therefore there are $\frac{n-2}{7}$minimal total unidominating functions.

Therefore there are $\frac{n-2}{7} + \frac{n-2}{7} = \frac{2n-4}{7} = \left\lfloor \frac{2n}{7} \right\rfloor$ minimal total unidominating functions with maximum weight$\left\lfloor \frac{5n}{7} \right\rfloor$.

Case 4: Let $n \equiv 3(mod\ 7)$.

A minimal total unidominating function $f$defined in Case 4 of Theorem 2.1 is given by



International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

The functional values of f are $0111110 \ldots 01111100111110011$.

Take $a - 0111110$. Then$f$ is in the pattern of $aaa \ldots aa011$. These letters can be arranged in only one way so that there is only one minimal total unidominating function.

Define another function $f_1: V \to \{0,1\}$ by

$$f_1(v_i) = \begin{cases} 1 & for\ i \equiv 0,1,2,5,6 (mod\ 7), \\ 0 & otherwise. \end{cases}$$

As above we can show that f_1 is a minimal total unidominating function.

$$\text{Now} \sum_{u \in V} f_1(u) = \underbrace{1+1+0}+\underbrace{0+1+1+1+1+1+0}+\cdots+$$

$$\underbrace{0+1+1+1+1+1+1+0}+\underbrace{0+1+1+1+1+1+0} = 2+\frac{5(n-3)}{7} = \frac{5n-1}{7}$$

$$= \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function is given by

The functional values of f_1 are $1100111110 \ldots 01111100111110$.

Take $a - 0111110$. Then the functional values of f_1 are in the pattern of $110aa \ldots aa$. These letters $aaa \ldots aa$ can be arranged in only one way.

Therefore there existsonly one function.

Thus there are only two minimal total unidominating functions with maximum weight$\left\lfloor \frac{5n}{7} \right\rfloor$.

Case 5: Let $n \equiv 4(mod\ 7)$.

A minimal total unidominating function f defined in Case 5 of Theorem 2.1 is given by

The functional values of f are $0111110 - - - 011111001111100110$.

International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Take a – 0111110, d – 0110. Then f is in the pattern of $aaa \ldots ad$. These letters $aaa \ldots ad$ can be arranged in $\frac{n-4}{7} + 1 = \frac{n+3}{7}$ ways.

Therefore there are $\frac{n+3}{7}$ minimal total unidominating functions.

As in Case 4 now we define another function $f_1 : V \to \{0,1\}$ by

$$f_1(v_i) = f(v_i), i \neq n-3, n-5,$$

and $f_1(v_{n-3}) = 1, f_1(v_{n-5}) = 0, n \geq 11.$

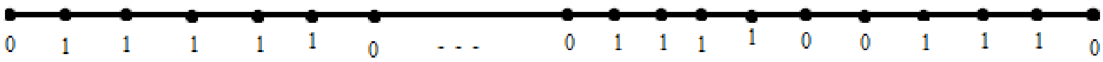
We can see that f_1 is a minimal total unidominating function.

Also

$$\sum_{u \in V} f_1(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+1+0}+$$

$$\underbrace{0+1+1+1+0} = \frac{5(n-11)}{7} + 4 + 3 = \frac{5n-6}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function is given by



The functional values of f_1 are 0111110 … 0111110011111001110.

Take a – 0111110, b – 011110, c – 01110, d – 0110. Then f_1 is in the pattern of $aa \ldots abc$. These $\frac{n+3}{7}$ letters $a's, b's$ and $c's$ can be arranged in

$$\frac{\left(\frac{n+3}{7}\right)!}{\left(\frac{n-11}{7}\right)!.1!.1!} = \binom{\frac{n+3}{7}}{}\binom{\frac{n-4}{7}}{} \text{ways.}$$

Therefore there are $\left(\frac{n+3}{7}\right)\left(\frac{n-4}{7}\right)$ minimal total unidominating functions.

Define another function $f_2 : V \to \{0,1\}$ by

$$f_2(v_i) = \begin{cases} 1 & for\ i \equiv 0,1,2,5,6 (mod\ 7), i \neq n-3, i \neq n-2, \\ 0 & for\ i \equiv 3,4\ (mod\ 7), i \neq n-1, i \neq n, \end{cases}$$

and$f_2(v_{n-3}) = 0, f_2(v_{n-2}) = 0, f_2(v_{n-1}) = 1, f_2(v_n) = 1, n \geq 11.$

We can see that this is a minimal total unidominating function.

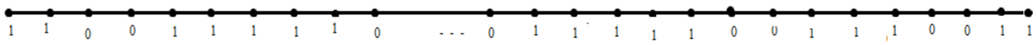
Also

$$\sum_{u \in V} f_2\,(u) = \underbrace{1+1+0}+\underbrace{0+1+1+1+1+1+0}+\cdots+$$

$$\underbrace{0+1+1+1+1+1+0}+\underbrace{0+1+1+1+0}\,+\underbrace{0+1+1} = 2+\frac{5(n-11)}{7}+3+2$$

$$= \frac{5n-6}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

The function f_2 is given by



The functional values of f_2 are $1100111110 \ldots 011111001110011$.

Take a – 0111110, c – 01110. Then f_2 is in the pattern of $110aa \ldots ac011$.

These $\frac{n-11}{7}$ a*'s and one* c can be arranged in $\frac{n-4}{7}$ ways.

Therefore there are $\frac{n-4}{7}$ minimal total unidominating functions.

Define another function $f_3 \colon V \to \{0,1\}$ by $f_3(v_i) = f_2(v_i),\ i \neq n-7, n-9$

and $f_3(v_{n-7}) = 1, f_3(v_{n-9}) = 0, n \geq 18.$

We can see that this is also a minimal total unidominating function.

Now

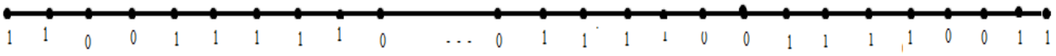
$$\sum_{u \in V} f_3\,(u) = \underbrace{1+1+0}+\underbrace{0+1+1+1+1+1+0}+\cdots+$$

$$\underbrace{0+1+1+1+1+0}+\underbrace{0+1+1+1+1+0}\,+\underbrace{0+1+1}$$

$$= 2+\left(\frac{5(n-18)}{7}\right)+4+4+2 = \frac{5n-6}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

This function is given by



The functional values of f_3 are $1100111110 \ldots 0111100111110011$.

Take $a - 0111110$, $b - 011110$. Then f_3 is in the pattern of $110aa \ldots abb011$. These $\frac{n-18}{7}$ a's and two b's can be arranged in $\frac{\left(\frac{n-4}{7}\right)!}{\left(\frac{n-18}{7}\right)!.2!} = \frac{\left(\frac{n-4}{7}\right).\left(\frac{n-11}{7}\right)}{2}$ ways.

Therefore there are $\frac{\left(\frac{n-4}{7}\right).\left(\frac{n-11}{7}\right)}{2}$ minimal total unidominating functions.

Define another function $f_4: V \to \{0,1\}$ by

$$f_4(v_i) = f_1(v_i), i \neq n-12, n-10, n-6, n-4,$$

and$f_4(v_{n-12}) = 0, f_4(v_{n-10}) = 1, f_4(v_{n-6}) = 0, f_4(v_{n-4}) = 1, n \geq 18$.

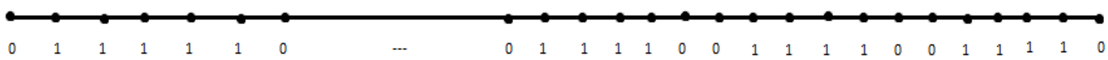
We can see that this is also a minimal total unidominating function.

Now $$\sum_{u \in V} f_4(u) = \underbrace{0+1+1+1+1+1+0}+\cdots+\underbrace{0+1+1+1+1+0}$$

$$+\underbrace{0+1+1+1+1+0}+\underbrace{0+1+1+1+1+0}$$

$$= \frac{5(n-18)}{7} + 4 + 4 + 4 + 4 = \frac{5n-6}{7} = \left\lfloor \frac{5n}{7} \right\rfloor.$$

This function is given by



The functional values of f_4 are $01111110 \ldots 0111100111100111100011110$.

Take $a - 0111110$, $b - 011110$. Then f_4 is in the pattern of $aa \ldots abbb$. These $\frac{n-18}{7}$ $a's$ and three b's can be arranged in $\frac{\left(\frac{n+3}{7}\right)!}{\left(\frac{n-18}{7}\right)!3!} = \frac{\left(\frac{n+3}{7}\right)\left(\frac{n-4}{7}\right)\left(\frac{n-11}{7}\right)}{3!}$ $ways.$

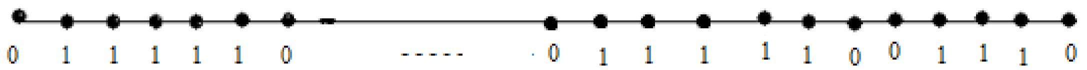
Therefore there are $\frac{\left(\frac{n+3}{7}\right)\left(\frac{n-4}{7}\right)\left(\frac{n-11}{7}\right)}{3!}$ minimal total unidominating functions.

Thus there are $\frac{n+3}{7} + \left(\frac{n+3}{7}\right)\left(\frac{n-4}{7}\right) + \frac{n-4}{7} + \frac{1}{2}\left(\frac{n-4}{7}\right)\left(\frac{n-11}{7}\right) + \frac{\left(\frac{n+3}{7}\right)\left(\frac{n-4}{7}\right)\left(\frac{n-11}{7}\right)}{3!}$

$= \frac{1}{2}\left\lfloor\frac{n}{7}\right\rfloor\left\lfloor\frac{3n}{7}\right\rfloor + \frac{1}{6}\left\lfloor\frac{n}{7}\right\rfloor\left\lfloor\frac{n}{7}\right\rfloor\left(\left\lfloor\frac{n}{7}\right\rfloor - 1\right)$ minimal total unidominating functions with maximum weight $\left\lfloor\frac{5n}{7}\right\rfloor$.

Case 6: Let $n \equiv 5(mod\ 7)$.

A minimal total unidominating function f defined as in Case 6 of Theorem 2.1 is given by



The functional values of f are 0111110 ... 011111001110.

Take a – 0111110, c – 01110. Then f is in the pattern of $aaa \ldots ac$. These $\frac{n-5}{7}$ $a's$ and one b can be arranged in $\frac{n+2}{7}$ ways.

Therefore there are $\frac{n+2}{7}$ minimal total unidominating functions.

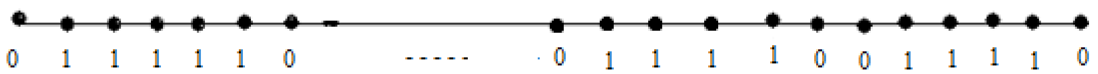
Define another function f_1 by

$$f_1(v_i) = f(v_i)\ for\ all\ v_i \in V, i \neq n-4, n-6$$

and $f_1(v_{n-4}) = 1, f_1(v_{n-6}) = 0, n \geq 12$.

We can see that this is a minimal total unidominating function and $f_1(V) = \left\lfloor\frac{5n}{7}\right\rfloor$.

This function is given by



The functional values of f_1 are 0111110 ... 011110011110.

Take a – 0111110, b – 011110. Then f_1 is in the pattern of $aaa \ldots abb$. Now there are $\frac{\left(\frac{n+2}{7}\right)!}{\left(\frac{n-12}{7}\right)!.2!} = \frac{\left(\frac{n+2}{7}\right)\left(\frac{n-5}{7}\right)}{2}$ minimal total unidominating functions.

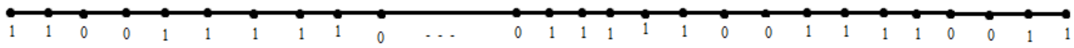
Another function $f_2 : V \to \{0,1\}$ is defined by

$$f_2(v_i) = \begin{cases} 1 & i \equiv 0,1,2,5,6(mod\ 7)\ i \neq n-3, \\ 0 & i \equiv 3,4(mod\ 7)\ i \neq n-1, \end{cases}$$

and $f_2(v_{n-3}) = 0, f_2(v_{n-1}) = 1$.

We can see that f_2 is a minimal total unidominating function and $f_2(V) = \left\lfloor \frac{5n}{7} \right\rfloor$.

This function is given by



The functional values of f_2 are 1100111110 … 0111110011110011.

Take a – 0111110, b – 011110. Then f_2 is in the pattern of $110aaa \ldots ab011$.

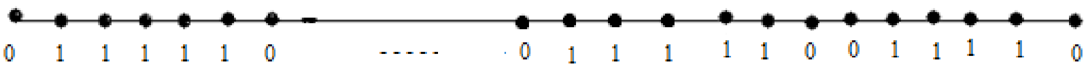
These $\frac{n-12}{7} + 1 = \frac{n-5}{7}$ letters can be arranged in $\frac{n-5}{7}$ ways.

Therefore there are $\frac{n-5}{7}$ minimal total unidominating functions.

Hence there are $\frac{n+2}{7} + \frac{1}{2}\left(\frac{n+2}{7}\right)\left(\frac{n-5}{7}\right) + \frac{n-5}{7} = \left\lceil \frac{n}{7} \right\rceil + \frac{1}{2}\left\lceil \frac{n}{7} \right\rceil \left\lfloor \frac{n}{7} \right\rfloor + \left\lfloor \frac{n}{7} \right\rfloor$ minimal total unidominating functions with maximum weight $\left\lfloor \frac{5n}{7} \right\rfloor$.

Case 7: Let $n \equiv 6(mod\ 7)$.

A minimal total unidominating function f defined as in Case 7 of Theorem 2.1 is given by



The functional values of f are 0111110 … 0111110011110.

International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Take $a - 0111110, b = 011110$.Then the function f is in the pattern of $aaa \dots ab$. As there are $\frac{n-6}{7} a's$ and one b, there exist $\frac{n-6}{7} + 1 = \frac{n+1}{7}$ minimal total unidominating functions.

Therefore the number of minimal total unidominating functions in the pattern of f are $\frac{n+1}{7}$.

Define another function $f_1 : V \to \{0,1\}$ by

$$f_1(v_i) = \begin{cases} 1 & for\ i \equiv 0,1,2,5,6 (mod\ 7), \\ 0 & otherwise. \end{cases}$$

We can easily verify that f_1 is a minimal total unidominating function.

The functional values of f_1 are $1100111110 \dots 0111110011$.

Take $a - 0111110$. Then f_1 is in the pattern of $110aaa \dots a011$.

This is the only one function in this pattern.

Therefore there are $\frac{n+1}{7} + 1 = \left\lceil \frac{n}{7} \right\rceil + 1$ minimal total unidominating functions with maximum weight $\left\lfloor \frac{5n}{7} \right\rfloor$. ■

3. ILLUSTRATIONS

Example 3.1: Let $n = 42$.

We know that $42 \equiv 0 (mod\ 7)$.

The functional values of a minimal total unidominating function f defined as in Case 1 of Theorem 2.1 for P_{42} are given at the corresponding vertices.

Upper total unidomination number $= \left\lfloor \frac{5 \times 42}{7} \right\rfloor = 30$. ■

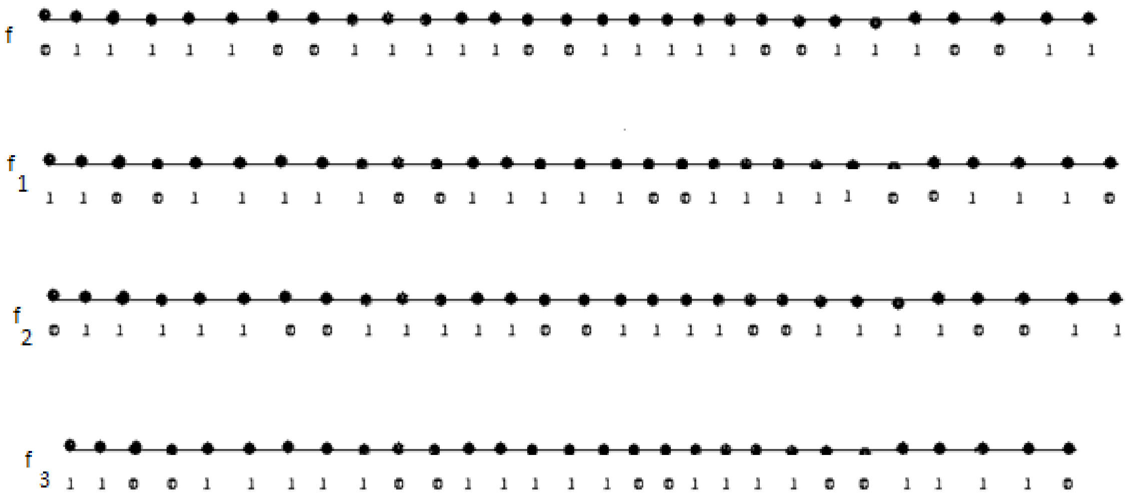
There is only one minimal total unidominating function for P_{42} with maximum weight.

Example 3.2: Let $n = 29$.

We know that $29 \equiv 1 (mod\ 7)$.

The functional values of minimal total unidominating functions f defined as in

Case 2 of Theorem 2.1 and f_1, f_2, f_3 defined as in Case 2 of Theorem 2.2for P_{29} are given at the corresponding vertices.



Upper total unidomination number ofP_{29} is $\left\lfloor \frac{5\ x\ 29}{7} \right\rfloor = 20$.

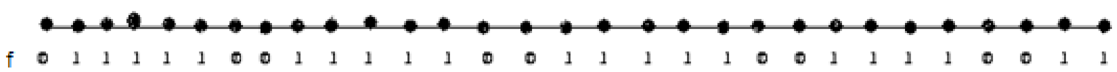
There are $\left\lfloor \frac{n}{7} \right\rfloor \left\lceil \frac{n}{7} \right\rceil = \left\lfloor \frac{29}{7} \right\rfloor \left\lceil \frac{29}{7} \right\rceil = 4x5 = 20$ minimal total unidominating functions with maximum weight. ■

Example 3.3: .Let $n = 30$.

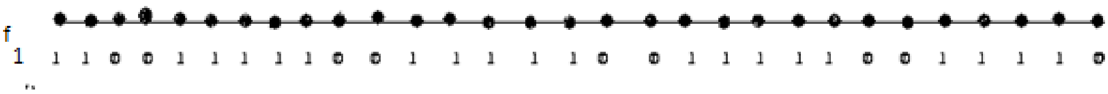
We know that $30 \equiv 2 (mod\ 7)$.

The functional values of minimal total unidominating functions f defined as in

Case 3 of Theorem 2.1 and f_1 defined as in Case 3 of Theorem 2.2 for P_{30} are given at the corresponding vertices.



International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A



Upper total unidomination number of P_{30} is $\left\lfloor\frac{5\ x\ 30}{7}\right\rfloor = 21.$

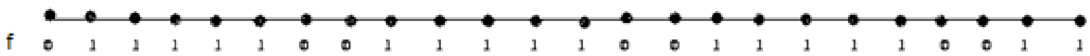
There are $\left\lfloor\frac{2n}{7}\right\rfloor = \left\lfloor\frac{2\ x\ 30}{7}\right\rfloor = 8$ minimal total unidominating functions with maximum weight. ■

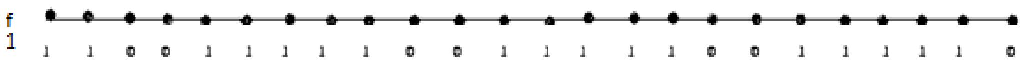
Example 3.4: Let $n = 24$.

We know that $24 \equiv 3(mod\ 7)$.

The functional values of a minimal total unidominating functions f defined as in

Case 4 of Theorem 2.1 and f_1 defined as in Case 4 of Theorem 2.2for P_{24} are given at the corresponding vertices.





Upper total unidomination number of P_{24} is $\left\lfloor\frac{5x24}{7}\right\rfloor = \left\lfloor\frac{120}{7}\right\rfloor = 17.$

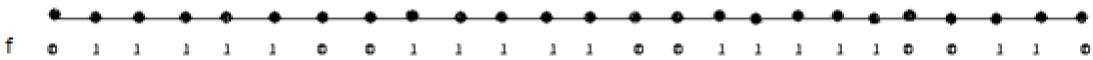
There are two minimal total unidominating functions with maximum weight. ■

Example 3.5: Let $n = 25$.

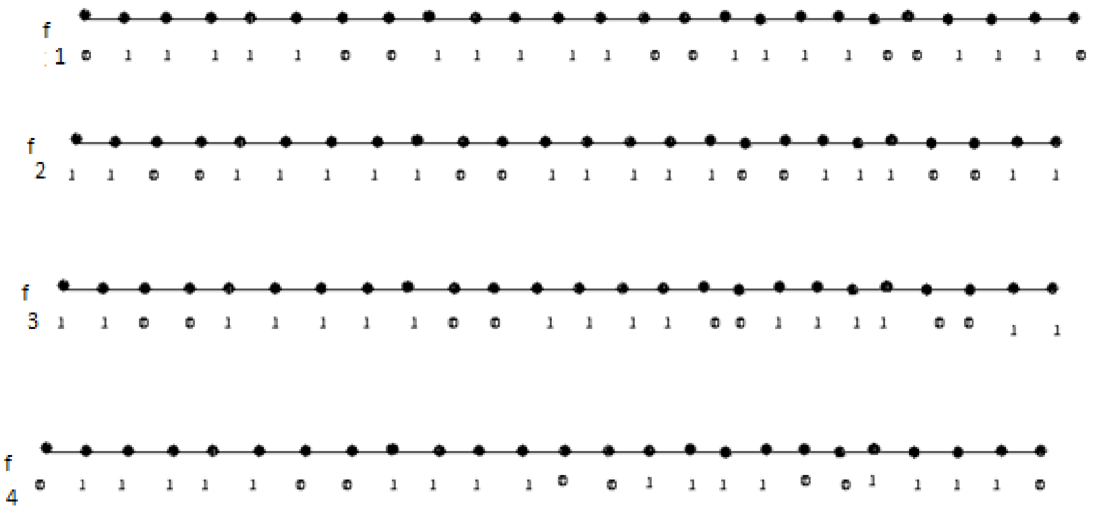
We know that $25 \equiv 4(mod\ 7)$.

The functional values of minimal total unidominating function f defined as in

Case 5 of Theorem 2.1 and f_1, f_2, f_3, f_4 defined as in Case 5 of Theorem 2.2 for P_{25} are given at the corresponding vertices.



International Journal of Engineering, Science and Mathematics
Vol.xIssue x, Month201x,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A



Upper total unidomination number is $\left\lfloor \frac{5 \text{ x } 25}{7} \right\rfloor = 17$.

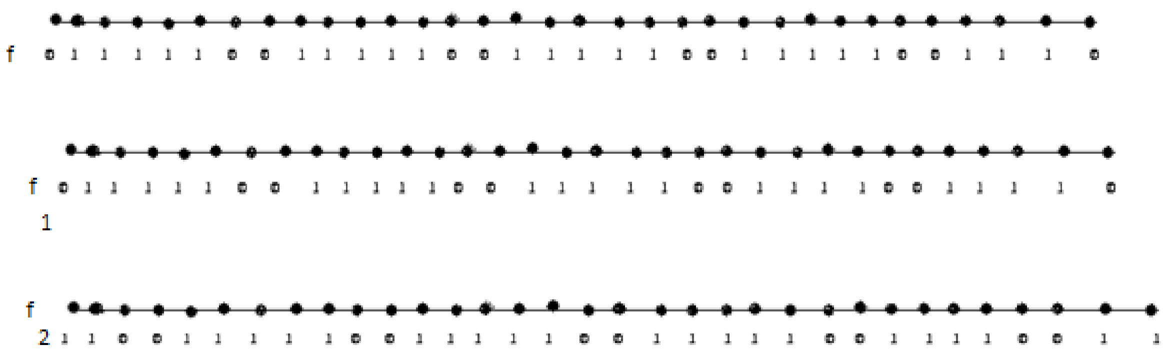
There are $\frac{1}{2}\left\lfloor \frac{n}{7} \right\rfloor \left\lfloor \frac{3n}{7} \right\rfloor + \frac{1}{6}\left\lfloor \frac{n}{7} \right\rfloor \left\lfloor \frac{n}{7} \right\rfloor \left(\left\lfloor \frac{n}{7} \right\rfloor - 1 \right) = 26$ minimal total unidominating functions with maximum weight. ■

Example 3.6: Let $n = 33$.

We know that $33 \equiv 5(mod\ 7)$.

The functional values of minimal total unidominating functions f defined as in

Case 6 of Theorem 2.1 and f_1, f_2 defined as in Case 6 of Theorem 2.2 for P_{33} are given at the corresponding vertices.



Upper total unidomination number is $\left\lfloor \frac{5 \text{ x } 33}{7} \right\rfloor = \left\lfloor \frac{165}{7} \right\rfloor = 23$.

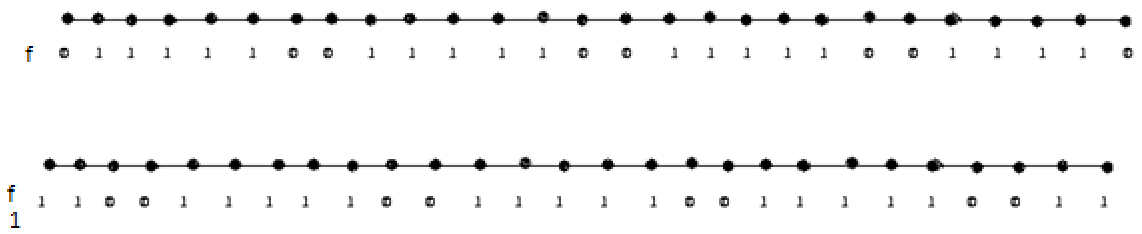
There are $\left\lceil\frac{33}{7}\right\rceil + \frac{1}{2}\left\lceil\frac{33}{7}\right\rceil\left\lfloor\frac{33}{7}\right\rfloor + \left\lfloor\frac{33}{7}\right\rfloor$ = 5+10+4 = 19 minimal total unidominating functions with maximum weight. ■

Example 3.7: Let $n = 27$.

We know that $27 \equiv 6(mod\ 7)$.

The functional values of minimal total unidominating functions f defined as in

Case 7 of Theorem 2.1 and f_1 defined as in Case 7 of Theorem 2.2 for P_{27} are given at the corresponding vertices.



Upper total unidomination number is $\left\lfloor\frac{5\ x\ 27}{7}\right\rfloor = 19$.

There are $\left\lceil\frac{27}{7}\right\rceil + 1 = 5$ minimal totalunidominating functions with maximum weight. ■

4. REFERENCES

1. Allan, R.B.Laskar, R.C.Hedetniemi, S.T. *A note on total domination*, Discrete Math. 49(1984), 7 – 13.

2. Berge, C., *The Theory of Graphs and its Applications*, Methuen, London (1962).

3. Cockayne, C.J.Dawes, R.M.Hedetniemi, S.T. - Total domination in graphs, Networks, 10 (1980), 211 – 219.

4. Cockayne, E.J.Mynhardt, C.M.Yu, B., *Total dominating functions in trees: Minimality and Convexity*, Journal of Graph Theory, 19(1995), 83 – 92.

5. Cockayne, E.J.Fricke, G.Hedetniemi, S.T.Mynhardt, C.M. - Properties of minimal dominating functions of graphs. ArsCombin., 41(1995), 107 – 115.

6. Hedetniemi, S.M.Hedetniemi, S.T.Wimer, T.V., *Linear time resource allocation algorithms for trees*. Technical report URI – 014, Department of Mathematics, Clemson University, (1987).

7. Ore, O., *Theory of Graphs*, Amer. Soc. Colloq. Publ. Vol.38. Amer. Math. Soc., Providence, RI, (1962).

8. Haynes, T.W.Hedetniemi, S.T.Slater, P.J. - Fundamentals of domination in graphs, Marcel Dekker, Inc., New York (1998).

9. Haynes, T.W.Hedetniemi, S.T.Slater, P.J. - Domination in Graphs : Advanced Topics, Marcel Dekker, Inc., New York (1998).

10. Anantha Lakshmi, V and Maheswari, B. – Total *Unidominating Functions of a Path*, IJCA, vol 126-No.13, (2015), 43-48.

11. V. Anantha Lakshmi, B. Maheswari - *Total Unidominating functions of a complete k-partite graph*: Open Journal of Applied & Theoretical Mathematics, Vol.2, No.4, December (2016) pp.795-805.

12. V. Anantha Lakshmi, B. Maheswari – Upper unidomination number of a path: IJRAR, Vol.5, Issue 4, November (2018) pp. 621-630

13. V. Anantha Lakshmi, B. Maheswari – Upper total unidomination number of a path: IJESM,Vol. 8 Issue 11, November(2019) pp. 44 - 59